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SANDWICH WING PANEL

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THE EFFECT OF TEMPERATURE STRESSES ON THE
STABILITY OF A SANDWICH WING PANEL

By

V. P. Karnozhitskiy

Translator's Note: In this article ch and sh in the
formulas correspond to cosh and sinh respectively.

THE EFFECT OF TEMPERATURE STRESSES ON THE STABILITY

OF A SANDWICH WING PANEL

V. P. Karnozhitskiy

This study examines the stability of a sandwich
wing panel compressed in one direction in the
presence of different temperatures on the upper
and lower carrying layers. It is considered that
the hypothesis of straight-line normals is valid
for the carrying layers. The general relations
of the theory of elasticity are used for the core
materials; however, it is assumed that the entire
compression load is received by the carrying layers
and that the core material up to the moment of
bulging is free of stresses. The temperature
stresses and strains in the core material are not
taken into account in view of the small rigidity of
the latter.

1. Statement of the Problem

During high-speed flights, the outer carrying layer of a sand-

wich wing panel located between the ribs and spars will be heated.

Since the inner carrying layer will have a lower temperature, tempera-

ture stresses will arise in the panel. Therefore, it is of interest

to elucidate how the temperature stresses acting in the panel will

affect the stability. As far as we know this article is the first

work devoted to this problem.

Let us state the problem in the following manner: to find the critical forces of a sandwich panel with a compressed load uniformly distributed along sides x = 0 and x = a (Fig. 1) if the temperature of the lower (inner) carrying layer is t_1, the temperature of the upper (outer) carrying layer is $t_2 > t_1$. The wing panel will be considered as a sandwich plate with unique boundary conditions. When we examine the temperature stresses to the right and left of the panel supports (ribs and spars), the conditions are completely identical; therefore the panel can be considered fastened on the supports. It is known that during loss of stability such a panel behaves as a freely resting plate.

Let us denote:

μ_f, E_f, a_f, δ the Poisson ratio, modulus of elasticity, coefficient of linear expansion, and the thickness of the carrying layer,

μ, E, G, $2h$ the Poisson ratio, modulus of elasticity, shear modulus, and the thickness of the core material.

2. Determination of Temperature Stresses in Carrying Layers

We will write the following known relations: for the lower carrying layer—

$$\varepsilon_{1x} = a_f t_1 + \frac{1}{E_f}(\sigma_{1x} - \mu_f \sigma_{1y}),$$

$$\varepsilon_{1y} = a_f t_1 + \frac{1}{E_f}(\sigma_{1y} - \mu_f \sigma_{1x}),$$

(2.1)

for the upper carrying layer—

$$\varepsilon_{2x} = a_f t_2 + \frac{1}{E_f}(\sigma_{2x} - \mu_f \sigma_{2y}),$$

$$\varepsilon_{2y} = a_f t_2 + \frac{1}{E_f}(\sigma_{2y} - \mu_f \sigma_{2x}),$$

where $\varepsilon_{1x}, \sigma_{1x}, \varepsilon_{1y}, \sigma_{1y}$ are the total strain and the temperature stresses

of the lower carrying layer along axes ox and oy,

$\varepsilon_{2x}, \sigma_{2x}, \varepsilon_{2y}, \sigma_{2y}$ are the total strain and temperature stresses of the

upper carrying layer along axes ox and oy.

Since no forces are applied to the plate from without (we consider only the effect of temperature), we can write that the total of all forces acting in the plate section by planes xoz and yoz equal zero:

$$2\delta a\,(\sigma_{1y} + \sigma_{2y}) = 0,$$
$$2\delta b\,(\sigma_{1x} + \sigma_{2x}) = 0.$$

$$(2.2)$$

In the equilibrium equations (2.2) we will disregard the stresses in the core material due to the small rigidity of the latter.

As was shown in my earlier study [1], a sandwich skin of a wing for practical purposes, does not bend in the presence of temperature stresses.* Therefore, we can write that the total strain of the carrying layers is identical:

$$\varepsilon_{1x} = \varepsilon_{2x}, \quad \varepsilon_{1y} = \varepsilon_{2y}.$$

$$(2.3)$$

Solving system of equations (2.1)-(2.3), we obtain the following expressions for temperature stresses:

$$\sigma_{1x} = \sigma_{1y} = -\sigma_{2x} = -\sigma_{2y} = \frac{\alpha_f E_f\,(t_2 - t_1)}{2\,(1 - \mu_f)}.$$

$$(2.4)$$

We will designate respectively by T_x and T_y the absolute magnitude of the linear temperature forces in the carrying layers. Then

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* The same as a one-layer fastened plate with a linear distribution of temperature throughout the thickness does not bend.

$$T_x = T_y = \frac{a_f E_f \delta \,(t_2 - t_1)}{2\,(1 - \mu_f)}.$$

<div align="right">(2.5)</div>

3. Stressed State of Carrying Layers

Let us separate from the upper carrying layer the element with sides dx and dy (Fig. 2).

We will introduce the designations:

$N_x,\ N_y$ linear tension forces,
$N_{xy},\ N_{yx},\ Q_x,\ Q_y$ linear shearing forces,
$M_x,\ M_y$ linear bending moments,
$M_{xy},\ M_{yx}$ twisting moments
$\sigma_z,\ \tau_{xz},\ \tau_{yz}$ components of stresses acting on the carrying layer from the side of the core material,
$u,\ v,\ w$ components of the displacements on the surface of contact between the carrying layer and core material.

The equilibrium equations of the upper carrying layer can be written as:

$$D\nabla^2\nabla^2 w - N_x \frac{\partial^2 w}{\partial x^2} - N_y \frac{\partial^2 w}{\partial y^2} - \sigma_z - \frac{\delta}{2}\left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y}\right) = 0$$

<div align="right">(3.1)</div>

at $z = -\,h$,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\delta}{2}\frac{\partial}{\partial x}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) + \frac{1}{2}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y} + \frac{\partial u}{\partial x}\right) + \frac{\mu_f}{2}\frac{\partial}{\partial y}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) + \frac{\tau_{xz}}{B} = 0$$

<div align="right">(3.2)</div>

at $z = -\,h$,

$$\frac{\partial^2 v}{\partial y^2} + \frac{\delta}{2}\frac{\partial}{\partial y}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) + \frac{1}{2}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) - $$
$$-\,\frac{\mu_f}{2}\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) + \frac{\tau_{yz}}{B} = 0,$$

<div align="right">(3.3)</div>

where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \quad D = \frac{E_f \delta^3}{12\,(1 - \mu_f^2)}, \quad B = \frac{E_f \delta}{1 - \mu_f^2}.$$

The equilibrium equations of the lower carrying layer are written analogously:

$$D\nabla^2\nabla^2 w - N_x \frac{\partial^2 w}{\partial x^2} - N_y \frac{\partial^2 w}{\partial y^2} + \sigma_z - \frac{\delta}{2}\left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y}\right) = 0$$

<div align="right">(3.4)</div>

at $z = h$,

<div align="center">-4-</div>

$$\frac{\partial^2 u}{\partial x^2} - \frac{\delta}{2}\frac{\partial}{\partial x}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) + \frac{1}{2}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) +$$
$$+ \frac{\mu_f}{2}\frac{\partial}{\partial y}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) - \frac{\tau_{xz}}{B} = 0, \tag{3.5}$$

at $z = h$,

$$\frac{\partial^2 v}{\partial y^2} - \frac{\delta}{2}\frac{\partial}{\partial y}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) + \frac{1}{2}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) -$$
$$- \frac{\mu_f}{2}\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) - \frac{\tau_{xz}}{B} = 0. \tag{3.6}$$

In our case

for the upper carrying layer

$$N_x = -P - T_x, \qquad N_y = -T_y, \tag{3.7}$$

for the lower carrying layer

$$N_x = -P + T_x, \qquad N_y = T_y, \tag{3.8}$$

where P is the linear force compressing the carrying layer towards ox.

Substituting (3.7) and (3.8) into (3.1) and (3.4), we obtain:

at $z = -h$

$$D\nabla^2\nabla^2 w + (P + T_x)\frac{\partial^2 w}{\partial x^2} + T_y\frac{\partial^2 w}{\partial y^2} - \sigma_z - \frac{\delta}{2}\left(\frac{\partial\tau_{xz}}{\partial x} + \frac{\partial\tau_{yz}}{\partial y}\right) = 0,$$

at
$$z = +h$$
$$D\nabla^2\nabla^2 w + (P - T_x)\frac{\partial^2 w}{\partial x^2} - T_y\frac{\partial^2 w}{\partial y^2} + \sigma_z - \frac{\delta}{2}\left(\frac{\partial\tau_{xz}}{\partial x} + \frac{\partial\tau_{yz}}{\partial y}\right) = 0.$$

4. Stressed State of Core Material

For any stressed state the components of stresses and strains can be expressed by the Maxwell formulas:

$$\sigma_x = \frac{\partial^2\eta}{\partial z^2} + \frac{\partial^2\zeta}{\partial y^2}, \qquad \tau_{yz} = -\frac{\partial^2\xi}{\partial y\,\partial z},$$

$$\sigma_y = \frac{\partial^2\zeta}{\partial x^2} + \frac{\partial^2\xi}{\partial z^2}, \qquad \tau_{xz} = -\frac{\partial^2\eta}{\partial x\,\partial z}, \tag{4.1}$$

$$\sigma_z = \frac{\partial^2\xi}{\partial y^2} + \frac{\partial^2\eta}{\partial x^2}, \qquad \tau_{xy} = -\frac{\partial^2\zeta}{\partial x\,\partial y},$$

$$2Gu = \frac{\partial}{\partial x}(\xi - \eta - \zeta), \tag{4.2}$$

$$2Gv = \frac{\partial}{\partial y}(-\xi + \eta - \zeta),$$

$$2Gw = \frac{\partial}{\partial z}(-\xi - \eta + \zeta).$$

where G is the shear modulus.

The biharmonic functions ξ, η, ζ must satisfy the following relations [2]

$$\nabla^2 \xi = \nabla^2 \eta = \nabla^2 \zeta,$$

$$(2-\mu)\nabla^2 \xi = \frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \eta}{\partial y^2} + \frac{\partial^2 \zeta}{\partial z^2},$$

(4.3)

where μ is the Poisson ratio,

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$$

If one of the components of the elementary rotation equals zero:

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0,$$

(4.4)

then

$$\xi(x, y, z) = \eta(x, y, z).$$

(4.5)

As A. P. Voronovich showed [3] the relation (44) for the core material is valid in the case of unidirectional compression of a freely resting sandwich plate. Let us assume the validity of (4.4) in our case. We will set the functions ξ, ζ in the form:

$$\xi = \eta = f_1(z) \sin \alpha x \sin \beta y,$$
$$\zeta = f_2(z) \sin \alpha x \sin \beta y,$$

(4.6)

where

$$\alpha = \frac{m\pi}{a}, \qquad \beta = \frac{n\pi}{b},$$

m and n are whole numbers.

The functions ξ and ζ are biharmonic, therefore having satisfied (4.3), we can write expressions (4.1) and (4.2) in the following manner:

$$2Gu = -\alpha \cos \alpha\, x \sin \beta\, y F_1(z), \quad \sigma_z = -\gamma^2 \sin \alpha\, x \sin \beta\, y F_3(z),$$
$$2Gv = -\beta \sin \alpha\, x \cos \beta\, y F_1\, z), \quad \tau_{xz} = -\alpha\gamma \cos \alpha\, x \sin \beta\, y F_4(z) \qquad (4.7)$$
$$2Gw = -\gamma \sin \alpha\, x \sin \beta\, y F_2(z), \quad \tau_{yz} = -\beta\gamma \sin \alpha\, x \cos \beta\, y F_4(z).$$

$$F_1(z) = c_3 \operatorname{ch} \gamma z + c_4 \operatorname{sh} \gamma z + c_5 \gamma z \operatorname{ch} \gamma z + c_6 \gamma z \operatorname{sh} \gamma z,$$
$$F_2(z) = (2c_1 - c_3)\operatorname{sh}\gamma z + (2c_2 - c_4)\operatorname{ch}\gamma z + c_5(\operatorname{ch}\gamma z + \gamma z \operatorname{sh}\gamma z) +$$
$$+ c_6(\operatorname{sh}\gamma z + \gamma z \operatorname{ch}\gamma z),$$
$$F_3(z) = c_1 \operatorname{ch}\gamma z + c_2 \operatorname{sh}\gamma z + c_5 \gamma z \operatorname{ch}\gamma z + c_6 \gamma z \operatorname{sh}\gamma z,$$
$$F_4(z) = c_1 \operatorname{sh}\gamma z + c_2 \operatorname{ch}\gamma z + c_5(\operatorname{ch}\gamma z + \gamma z \operatorname{sh}\gamma z) +$$
$$+ c_6(\operatorname{sh}\gamma z + \gamma \dot z \operatorname{ch}\gamma z), \qquad (4.8)$$

where
$$\gamma^2 = \alpha^2 + \beta^2,$$

c_1, c_2, c_3, c_4, c_5, c_6 are arbitrary constants satisfying equations:

$$c_1 - c_3 + 2(1 - \mu)c_6 = 0, \qquad (4.9)$$
$$c_2 - c_4 + 2(1 - \mu)c_5 = 0.$$

5. Simultaneous Deformation of the Carrying Layers and Core Material. Expression for the Critical Force

In order to determine the arbitrary constants we will use equations (3.2), (3.3), (3.5), (3.6), and (3.9), substituting there the expressions (4.7) when $z = \pm h$. After simple transformation these equations* can be written as:

$$\left(-\delta\gamma^2 + \frac{2G}{B}\right)\operatorname{sh}\gamma h \cdot c_1 + \gamma\left(\operatorname{ch}\gamma h + \frac{\delta\gamma}{2}\operatorname{sh}\gamma h\right)c_3 +$$
$$+\left[\gamma^2 h \operatorname{sh}\gamma h + \left(-0.5\delta\gamma^2 + \frac{2G}{B}\right)(\operatorname{sh}\gamma h + \gamma h \operatorname{ch}\gamma h)\right]c_6 = 0,$$

$$\left(-\delta\gamma^2 + \frac{2G}{B}\right)\operatorname{ch}\gamma h \cdot c_2 + \left(\gamma \operatorname{sh}\gamma h + \frac{\delta\gamma^2}{2}\operatorname{ch}\gamma h\right)c_4 +$$
$$+\left[\gamma^2 h \operatorname{ch}\gamma h + \left(-0.5\delta\gamma^2 + \frac{2G}{B}\right)(\operatorname{ch}\gamma h + \gamma h \operatorname{sh}\gamma h)\right]c_5 = 0,$$

$$[(0.5\delta\gamma^2 + 2A_1)\operatorname{sh}\gamma h + \gamma \operatorname{ch}\gamma h]c_1 - A_1 \operatorname{sh}\gamma h \cdot c_3 - A_2 \operatorname{ch}\gamma h \cdot c_4 +$$
$$+ [\gamma^2 h \operatorname{sh}\gamma h + (A_1 + 0.5\delta\gamma^2)(\operatorname{sh}\gamma h + \gamma h \operatorname{ch}\gamma h)]c_6 + 2A_2 \operatorname{ch}\gamma h c_2 +$$

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* Two of which are the consequence of two others.

$$+ A_2 (\text{ch } \gamma h + \gamma h \text{ sh } \gamma h) c_5 = 0,$$

$$2A_2 \text{ sh } \gamma h \cdot c_1 - A_2 \text{ sh } \gamma h \cdot c_3 + A_2 (\text{sh } \gamma h + \gamma h \text{ ch } \gamma h) c_6 +$$

$$+ [\gamma \text{ sh } \gamma h + (2A_1 + 0.5 \delta \gamma^2) \text{ ch } \gamma h] c_2 - A_1 \text{ ch } \gamma h \cdot c_4 +$$

$$+ [(A_1 + 0.5 \delta \gamma^2)(\text{ch } \gamma h + \gamma h \text{ sh } \gamma h) + \gamma^2 h \text{ ch } \gamma h] c_5 = 0, \qquad (5.1)$$

where

$$A_1 = \frac{D\gamma^4 - P\alpha^2}{2G},$$

$$A_2 = \frac{T_x \alpha^2 + T_y \beta^2}{2G}.$$

The system of homogeneous equations (5.1) and (4.9) yields values of c_1, c_2, c_3, c_4, c_5, c_6 which are nonzero if the determinant equals zero:

$\left(\frac{2G}{B} - \delta\gamma^2\right) \times$ $\times \text{sh } \gamma h$	$\gamma \text{ ch } \gamma h +$ $+ \frac{\delta\gamma^2}{2} \text{ sh } \gamma h$	$\gamma h \text{ sh } \gamma h +$ $+ \left(\frac{2G}{B} - \frac{\delta\gamma^2}{2}\right) \times$ $\times (\text{sh}\gamma h + \gamma h \text{ch } \gamma h)$	0	0	0
0	0	0	$\left(\frac{2G}{B} - \delta\gamma^2\right) \times$ $\times \text{ch } \gamma h$	$\gamma \text{ sh } \gamma h +$ $+ \frac{\delta\gamma^2}{2} \text{ch}\gamma h$	$\gamma^2 h \text{ ch } \gamma h +$ $+ \left(\frac{2G}{B} - \frac{\delta\gamma^2}{2}\right) \times$ $\times (\text{ch } \gamma h + \gamma h \text{ sh}\gamma h)$
$\gamma \text{ ch } \gamma h +$ $+ \left(\frac{\delta\gamma^2}{2} + 2A_1\right) \times$ $\times \text{sh } \gamma h$	$- A_1 \text{ sh } \gamma h$	$\gamma^2 h \text{ sh } h\gamma h +$ $+ \left(\frac{\delta\gamma^2}{2} + A_1\right) \times$ $\times \text{ sh}\gamma h + \gamma h\text{ch}_1 h)$	$2A_2 \text{ ch } \gamma h$	$-A_2\text{ch}\gamma h$	$A_2 (\text{ch } \gamma h +$ $+ \gamma h \text{ sh } \gamma h)$
$2A_2 \text{ sh } \gamma h$	$- A_2 \text{ sh } \gamma h$	$A_2 (\text{sh } \gamma h +$ $+ \gamma h \text{ ch } \gamma h)$	$\gamma \text{ sh } \gamma h +$ $+ \left(\frac{\delta\gamma^3}{2} + 2A_1\right) \times$ $\times \text{ch } \gamma h$	$-A_1\text{ch}\gamma h$	$\gamma^2 h \text{ ch } \gamma h +$ $+ \left(A_1 + \frac{\delta\gamma^2}{2}\right) \times$ $\times (\text{ch } \gamma h + \gamma h \text{ sh}\gamma h)$
1	-1	$2(1-\mu)$	0	0	0
0	0	0	1	-1	$2(1-\mu)$

$$= 0 \qquad (5.2)$$

After a series of transformations, equation (5.2) can be written in the form:

$$(P_c - P)(P_a - P) = \left(T_x + T_y \frac{\beta^2}{a^2} \right)^2,$$

(5.3)

$$P_c = \frac{D\gamma^4}{a^2} + \frac{2G\gamma}{a^2} \cdot \frac{\dfrac{(1-\mu)\delta^2\gamma^2}{2}\, \text{sh}^2\,\gamma h + \left[(1-2\mu)\delta\gamma + \dfrac{2G}{\gamma B}\right]\text{sh}\,\gamma h\,\text{ch}\,\gamma h +}{\dfrac{4(1-\mu)G}{\gamma B}\,\text{sh}^2\,\gamma h + (3-4\mu)\,\text{sh}\,\gamma h\,\text{ch}\,\gamma h - \gamma h} +$$

$$+ \frac{2G\gamma}{a^2} \cdot \frac{+ 2(1-\mu)\,\text{ch}^2\,\gamma h - \delta h\gamma^2 + \dfrac{2G}{B}\,h}{\dfrac{4(1-\mu)G}{\gamma B}\,\text{sh}^2\,\gamma h + (3-4\mu)\,\text{sh}\,\gamma h\,\text{ch}\,\gamma h - \gamma h},$$

(5.4)

$$P_a = \frac{D\gamma^4}{a^2} + \frac{2G\gamma}{a^2} \cdot \frac{2(1-\mu)\,\text{sh}^2\,\gamma h + \dfrac{1}{2}(1-\mu)\delta^2\gamma^2\,\text{ch}^2\,\gamma h +}{(3-4p)\,\text{sh}\,\gamma h\,\text{ch}\,\gamma h + \dfrac{4(1-\mu)G}{B\gamma}\,\text{ch}^2\,\gamma h + \gamma h} +$$

$$+ \frac{2G\gamma}{a^2} \cdot \frac{+\left[\delta\gamma(1-2\mu) + \dfrac{2G}{\gamma B}\right]\text{sh}\,\gamma h\,\text{ch}\,\gamma h - \dfrac{2G}{B}\,h + \delta h\gamma^2}{(3-4\mu)\,\text{sh}\,\gamma h\,\text{ch}\,\gamma h + \dfrac{4(1-\mu)G}{B\gamma}\,\text{ch}^2\,\gamma h + \gamma h},$$

(5.5)

P–linear critical force of one carrying layer (half the linear critical force of the entire sandwich layer) which hereafter we will call the critical load,

P_c and P_a—critical loads of symmetric and antisymmetric forms of stability loss for a freely resting sandwich plate compressed in one direction, which were obtained by A. P. Voronovich [3].

From (5.3) we obtain two values of the critical load:

$$P_{-,+} = \frac{P_c + P_a}{2} \pm \sqrt{\frac{(P_c - P_a)^2}{4} + \left(T_x + T_y \frac{\beta^2}{a^2} \right)^2}$$

(5.6)

or, taking into account (2.5)

$$P_{-,+} = \frac{P_c + P_a}{2} \pm \sqrt{\frac{(P_c - P_a)^2}{4} + \left[\frac{a_f E_f \delta\,(t_2 - t_1)}{2(1-\mu_f)} \left(1 + \frac{\beta^2}{a^2}\right)\right]^2},$$

(5.7)

where α and β are found from the condition of the minimum* for P.

---

* It is necessary to understand that α and β can take on values corresponding only to integral values of n and m.

Of practical interest is the smaller critical value:

$$P_- = \frac{P_c + P_a}{2} - \sqrt{\frac{(P_c - P_a)^2}{4} + \left(T_x + T_y \frac{\beta^2}{a^2}\right)^2}. \tag{5.8}$$

6. Particular Cases. Analysis of the Formulas Derived

If the temperature of the carrying layers is identical ($t_1 = t_2$) we obtain from (5.7):

$$P_- = P_a, \qquad P_+ = P_c,$$

i.e., complete agreement with the results of A. P. Voronovich [3].

Since the second term of the radicand in formula (5.8) is always positive:

$$P_- < P_a, \tag{6.1}$$

i.e., the presence of temperature stresses reduces the critical forces of the plate.

For real sandwich panels used in aircraft construction, always $P_a < P_c$. Consequently,

$$\sqrt{\frac{(P_c - P_a)^2}{4} + \left(T_x + T_y \frac{\beta^2}{a^2}\right)^2} < \frac{P_c - P_a}{2} + T_x + T_y \frac{\beta^2}{a^2}. \tag{6.2}$$

Substituting (6.2) into (5.8), we obtain:

$$P_- > P_a - \left(T_x + T_y \frac{\beta^2}{a^2}\right). \tag{6.3}$$

If for an infinitely wide plate ($b = \infty$) we assume $T_y = 0$, formula (6.3) has the form:

$$P_- > P_a - T_x,$$

i.e., the critical stresses are reduced by a magnitude smaller than the magnitude of the temperature stresses.*

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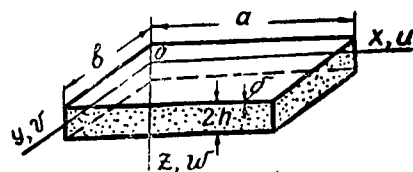* Physically this is explained by tensile temperature stresses in the lower carrying layer.
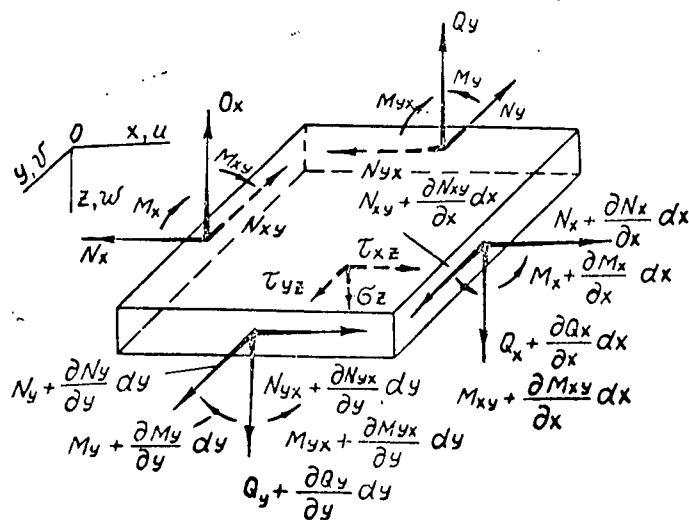
Fig. 1.

Fig. 2.

Under these assumptions, we obtain from (5.3):

$$(P_c - P)(P_a - P) = T_x^2.$$

Cox [4]* obtained an analogous expression; he examined the stability of a freely resting sandwich rod under the following load: one carrying layer is compressed by linear forces $P + T_x$ and the other by forces $P - T_x$.

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Submitted

October 18, 1960.

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* The results of study [4] are given in [5].